An expression for the field dependence of Zernike polynomials in rotationally symmetric optical systems

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Background

Traditionally, testing, design and alignment of optical imaging systems, including space telescopes, have relied heavily on interferograms to provide pupil dependence of the wavefront aberration function, W.

However, this is only half the available information that can be obtained from the wavefront aberration function, it being a function of pupil *and field* parameters.

Potential Benefits
- testing procedures
- new design strategies/tools
- alignment procedures

Methodology

Utilizing pupil and field dependence will require

- an analytic equation for W that provides explicit field dependence of the expansion coefficient functions, and
- new ways of displaying the field dependence of the aberration function.

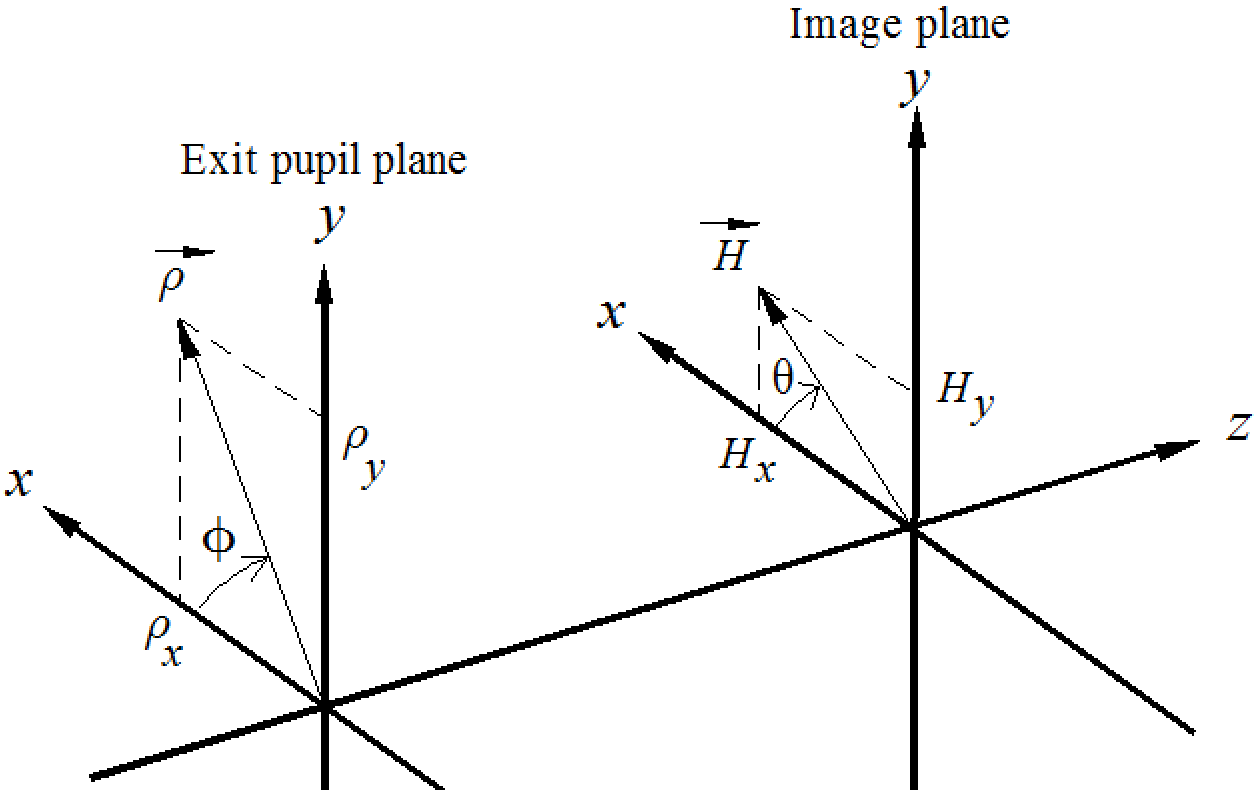
Outline

- Derive the wavefront aberration function for rotationally symmetric imaging systems in terms of Zernike polynomials
- Show explicitly the field dependence of the expansion coefficient functions
- Qualitative validation of the equations using Matlab® and real raytracing from CODE V® for a JWST-like model (design data from SPIE 2004)

Coordinate System

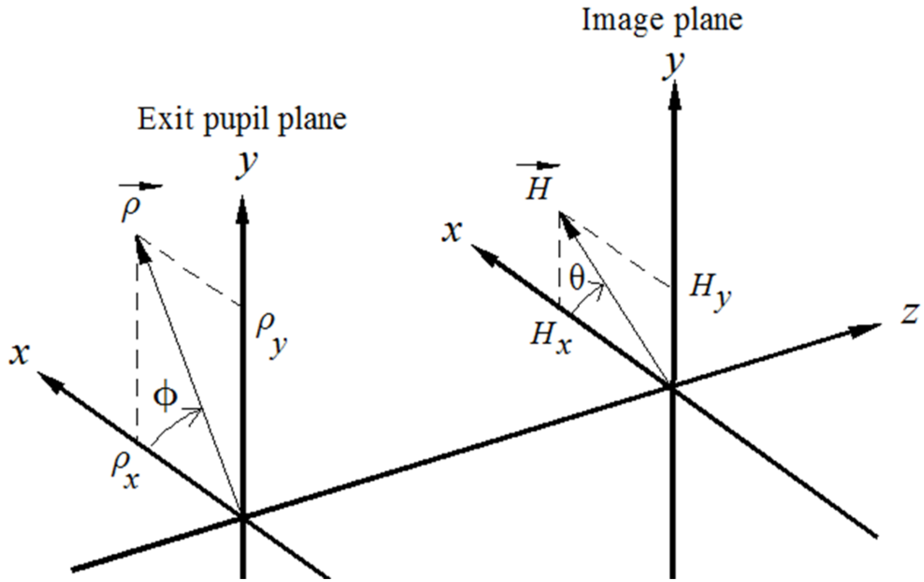
The coordinate system:

- Right-handed coordinate system
- Optical axis = +*z*-axis, direction of propagation
- Angles measured from +x-axis



Aberration Function

A familiar form (modified)



$$W\left(\vec{H},\vec{\rho}\right)=\sum_{j}\sum_{p}\sum_{n}\sum_{m}W_{klm;j}H^{k}\rho^{l}\cos^{m}\left(\theta-\phi\right)$$

$$k=2p+m,\quad l=2n+m$$

We do not assume H is aligned along the +x-axis.

We have also implicitly normalized H and ρ per surface j.

Through 5th Order Plus 7th Spherical

Dropping the sum over each surface for the moment,

$$\begin{aligned}W = {} & \Delta W_{20}\rho^2 + \Delta W_{11} H\rho\cos(\theta-\phi) + W_{040}\rho^4 + W_{131} H\rho^3\cos(\theta-\phi) \\ & + W_{220S} H^2\rho^2 + W_{222} H^2\rho^2\cos^2(\theta-\phi) + W_{311} H^3\rho\cos(\theta-\phi) \\ & + W_{060}\rho^6 + W_{151} H\rho^5\cos(\theta-\phi) + W_{240S} H^2\rho^4 + W_{242} H^2\rho^4\cos^2(\theta-\phi) \\ & + W_{331S} H^3\rho^3\cos(\theta-\phi) + W_{333} H^3\rho^3\cos^3(\theta-\phi) + W_{420S} H^4\rho^2 \\ & + W_{422} H^4\rho^2\cos^2(\theta-\phi) + W_{511} H^5\rho\cos(\theta-\phi) + W_{080}\rho^8\end{aligned}$$

We wish to convert this to an expansion in Zernike polynomials of the pupil coordinates ρ and ϕ. This will separate the field angle from the pupil angle and convert $\cos^2()$ and $\cos^3()$ functions to cos(2_) and cos(3_) functions, respectively.

Zernike Polynomials

We use standard Born & Wolf Zernike polynomials because

- They are orthogonal and complete over a unit radius circular aperture (exit pupil)
- They are familiar to the various optics communities
- There are established methods for finding the expansion coefficients

$$Z_n^{\pm m}(\rho,\phi) = R_n^m(\rho)\begin{cases}\cos(m\phi) & \mathit{for}\ +m \\ \sin(m\phi) & \mathit{for}\ -m\end{cases}$$

$$R_n^m(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s!\left(\frac{n+m}{2}-s\right)!\left(\frac{n-m}{2}-s\right)!}\rho^{n-2s}$$

$$N_{nm} = \left|Z_n^{\pm m}(\rho,\phi)\right|^2 = \int_0^{2\pi}\int_0^1 Z_n^{\pm m}(\rho,\phi)\, Z_n^{\pm m}(\rho,\phi)\,\rho\, d\rho\, d\phi = \frac{\pi\left(1+\delta_{0m}\right)}{2(n+1)}$$

Obtain Expansion Coefficient Functions

$$\begin{aligned} W = \cdots &+ A_2^2\left[Z_2^2(\rho,\phi)\right] + A_2^{-2}\left[Z_2^{-2}(\rho,\phi)\right] && \{Astigmatism\} \\ &+ A_3^1\left[Z_3^1(\rho,\phi)\right] + A_3^{-1}\left[Z_3^{-1}(\rho,\phi)\right] && \{Coma\} \\ &+ A_4^0\left[Z_4^0(\rho,\phi)\right] + \cdots && \{Spherical\} \end{aligned}$$

To obtain the Zernike expansion coefficients *A*, we use the orthogonality of the Zernike polynomials.

$$A_n^{\pm m}(H,\theta) = \frac{1}{N_{nm}} \int_0^{2\pi}\int_0^1 W(H,\theta,\rho,\phi)\, Z_n^{\pm m}(\rho,\phi)\, \rho\, d\rho\, d\phi$$

A's are functions of the field parameters.

Field Dependent Zernike Expansion

For Zernike Astigmatism, Coma and Spherical aberrations (<u>in pupil</u> parameters) we obtain

$$
\begin{aligned}
W = \cdots &+ \left[\left(\frac{1}{2} W_{222} + \frac{3}{8} W_{242} + \frac{1}{2} W_{422} H^2 \right) H^2 \cos(2\theta) \right] \left[Z_2^2(\rho,\phi) \right] \\
&+ \left[\left(\frac{1}{2} W_{222} + \frac{3}{8} W_{242} + \frac{1}{2} W_{422} H^2 \right) H^2 \sin(2\theta) \right] \left[Z_2^{-2}(\rho,\phi) \right] \quad \{Astigmatism\} \\
&+ \left[\left(\frac{1}{3} W_{131} + \frac{2}{5} W_{151} + \frac{1}{3} W_{331M} H^2 \right) H \cos(\theta) \right] \left[Z_3^1(\rho,\phi) \right] \\
&+ \left[\left(\frac{1}{3} W_{131} + \frac{2}{5} W_{151} + \frac{1}{3} W_{331M} H^2 \right) H \sin(\theta) \right] \left[Z_3^{-1}(\rho,\phi) \right] \quad \{Coma\} \\
&+ \left[\frac{1}{6} W_{040} + \frac{1}{4} W_{060} + \frac{2}{7} W_{080} + \frac{1}{6} W_{240M} H^2 \right] \left[Z_4^0(\rho,\phi) \right] + \cdots \quad \{Spherical\}
\end{aligned}
$$

Use Matlab® & CODE V® For Validation

A Matlab® program was written to plot the field dependence of the Zernike expansion coefficient functions (not the Zernike polynomials) using the Zernike equation for W.

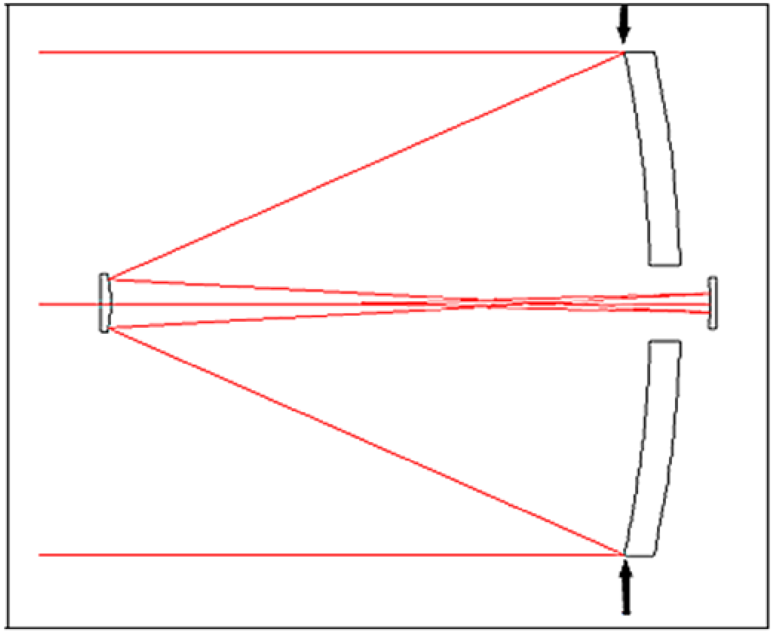
We used CODE V® macros fifthdef.seq and forder.seq to obtain the W_{klm}-coefficients.

And we used CODE V® (FMA; FFD) to produce real raytrace data and plots of the field dependence of the aberrations.

A qualitative comparison between the plots was performed.

An Example

One of the CODE V® models we used was an early version of a James Webb Space Telescope-like (JWST) model.



(Arrows indicate aperture stop.)

- 3 mirror telescope
- Field biased
- New look using the new field oriented tools

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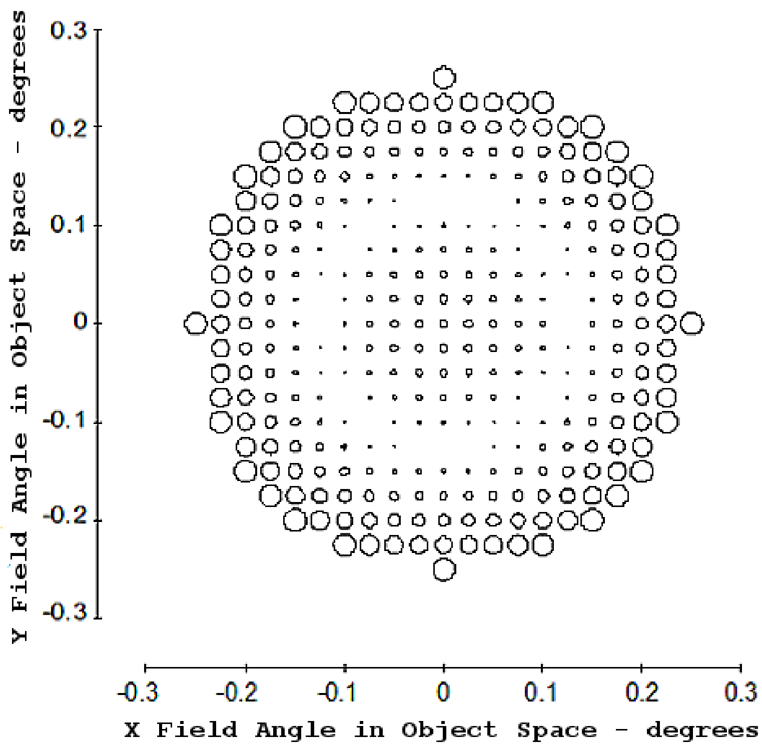
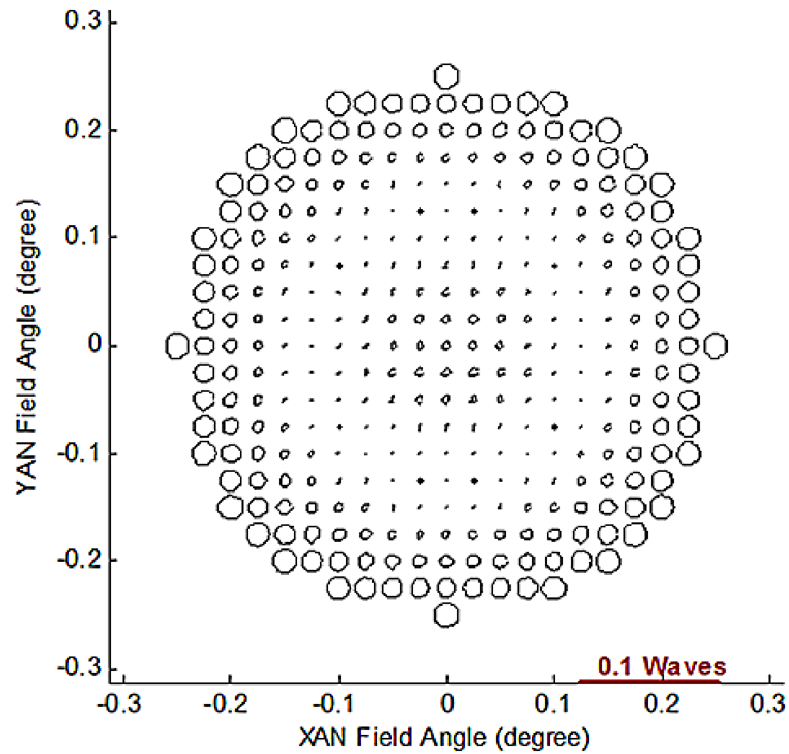
JWST: Zernike Spherical (Field Function Plot)

$$\left[\frac{1}{6}W_{040}+\frac{1}{4}W_{060}+\frac{2}{7}W_{080}+\frac{1}{6}W_{240M}H^{2}\right]\left[Z_{4}^{0}\left(\rho,\phi\right)\right]$$





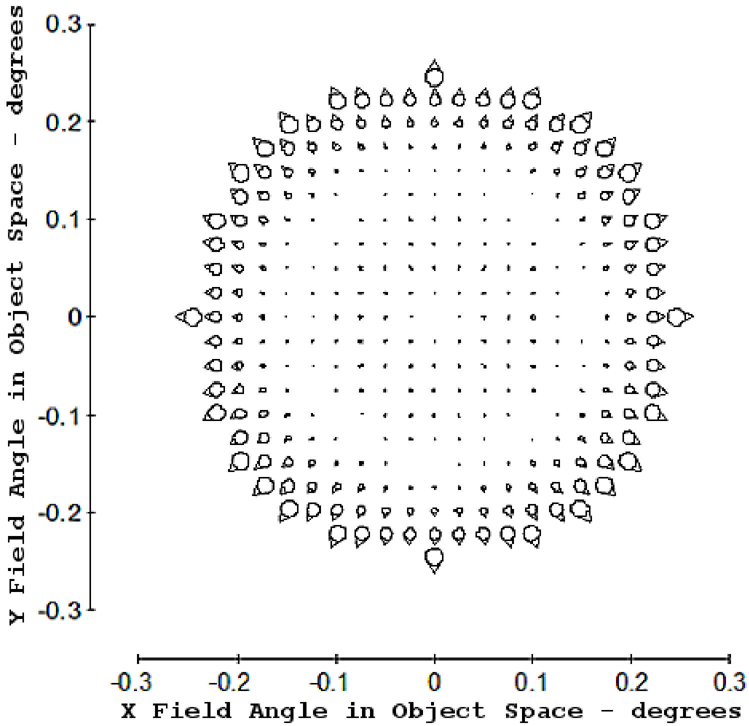
CODE V®
Real raytracing plus fitting

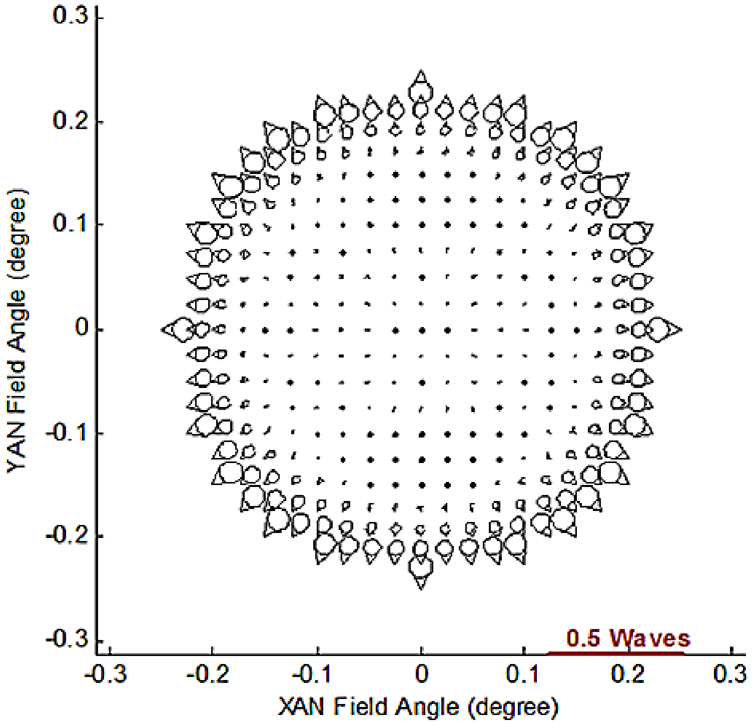
Analytical equations
Matlab® program

We learn that Zernike spherical depends on field. Use this in design.

JWST: Zernike Coma (Field Function Plot)

$$\left[\left(\frac{1}{3}W_{131}+\frac{2}{5}W_{151}+\frac{1}{3}W_{331M}H^{2}\right)H\cos(\theta)\right]\left[Z_{3}^{1}(\rho,\phi)\right]$$





CODE V®
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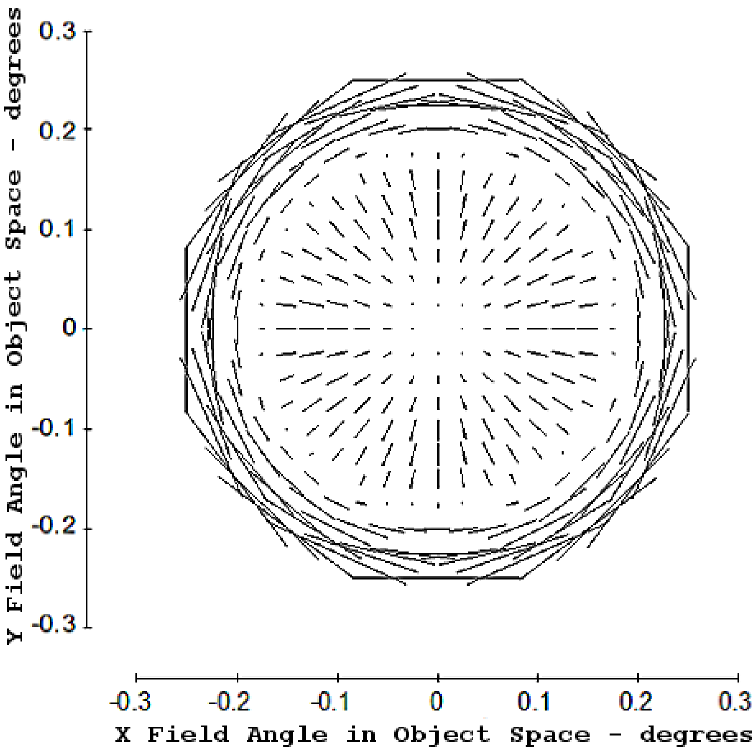
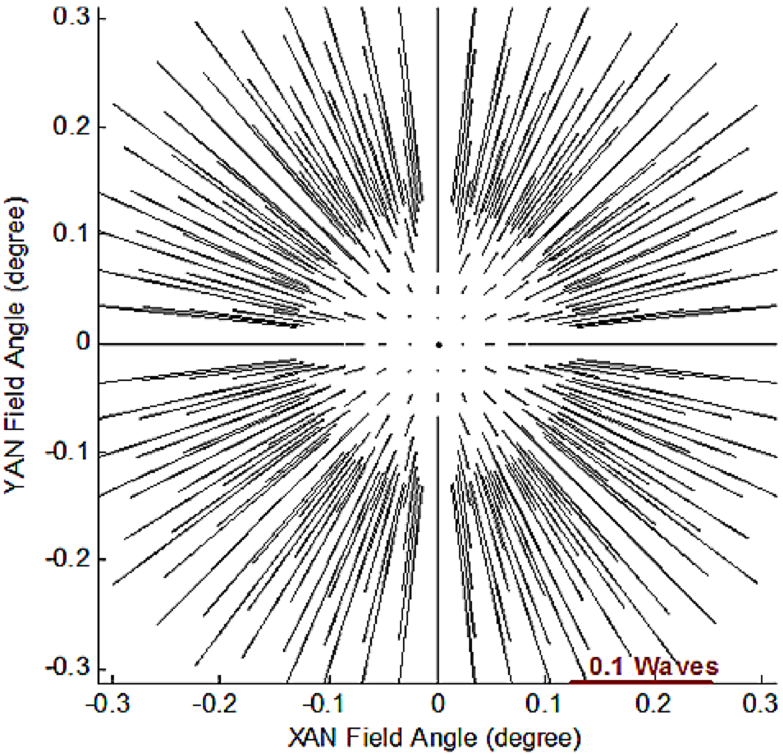
JWST: Zernike Astigmatism (Thru 5th order)

$$\left[\left(\frac{1}{2}W_{222}+\frac{3}{8}W_{242}+\frac{1}{2}W_{422}H^{2}\right)H^{2}\cos\left(2\theta\right)\right]\left[Z_{2}^{2}\left(\rho,\phi\right)\right]$$



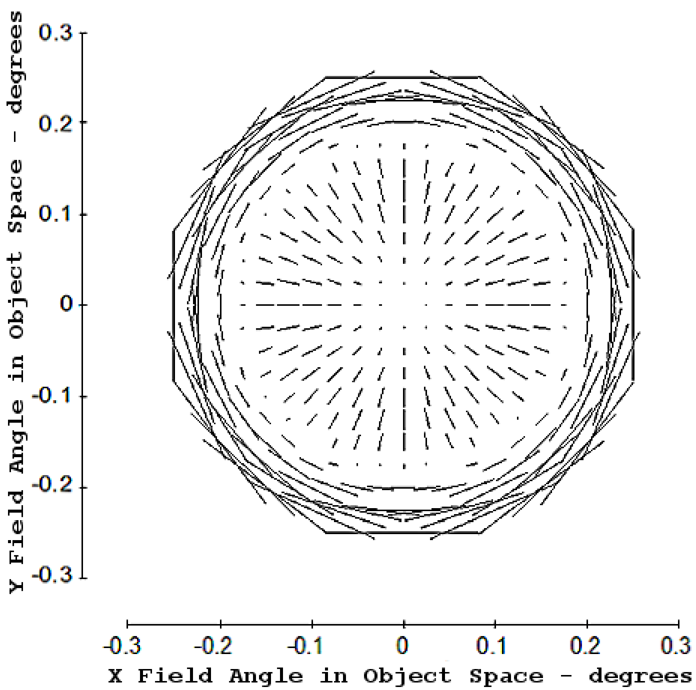


CODE V®
Real raytracing plus fitting

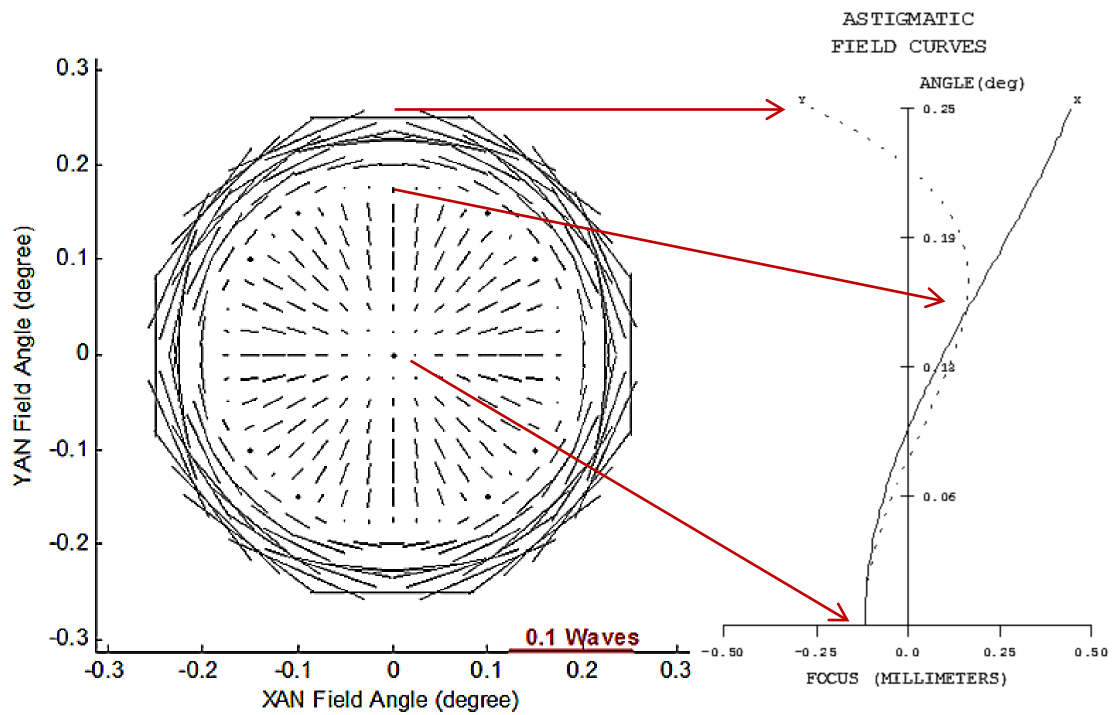
Analytical equations
Matlab® program

JWST: Zernike Astigmatism (Thru 7th order)

$$\left[\left(\frac{1}{2}W_{222}+\frac{3}{8}W_{242}+\underline{\frac{3}{10}W_{262}}+\left(\frac{1}{2}W_{422}+\underline{\frac{3}{8}W_{442}+\frac{3}{8}W_{444}}\right)H^{2}+\underline{\frac{1}{2}W_{622}H^{4}}\right)H^{2}\cos(2\theta)\right]\left[Z_{2}^{2}(\rho,\phi)\right]$$



CODE V®
Real raytracing plus fitting

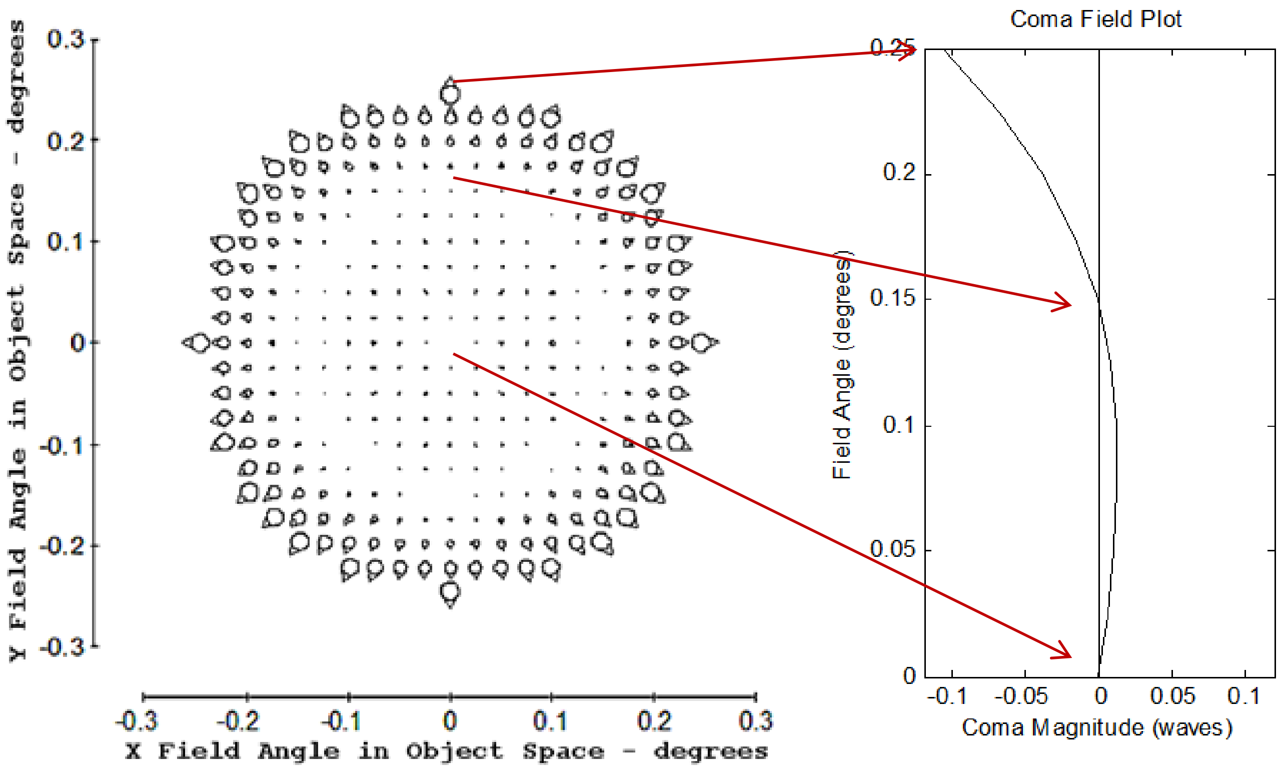


Analytical equations
Matlab® program

We learn that JWST is limited by 7th order astigmatism, not 5th order.

Design Tools

The field curves for astigmatism is an important tool in design. Calculated using Coddington's equation, is fast. We'd like a similar tool for displaying the field dependence of coma. The developed field equations is a step in this direction.



Conclusion

We have developed the explicit field dependence of the Zernike expansion coefficient functions for the wavefront aberration function.

These equations are in good qualitative agreement with the results from real raytracing optical design software.

Developing new tools that allow us to more fully utilize field dependence in design.

The next step is to explore extensions of the field dependence equations to include decenters and tilts in rotationally symmetric imaging optical systems.

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